IMACS

INGÉNIERIE MATHÉMATIQUE

& CALCUL SCIENTIFIQUE

RETARDED POTENTIAL METHOD FOR NON-HOMOGENEOUS MEDIA AND APPROXIMATION OF THIN LAYERS BY GENERALIZED IMPEDANCE CONDITIONS IN ELECTRO-MAGNETICS.

Toni SAYAH

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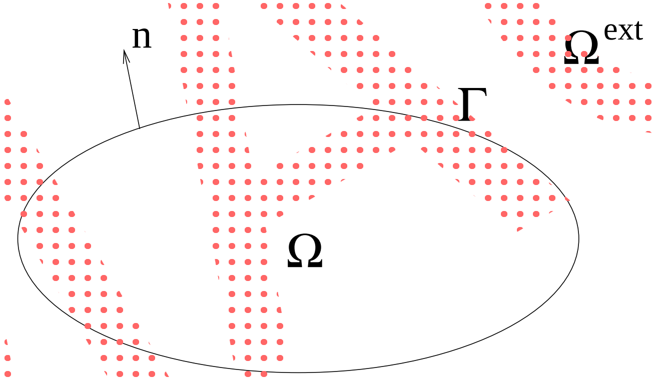
1. Introduction : Perfect conductor case

(Agnès Pujols, Isabelle Terrasse and Virginie Lange)



a. Maxwell Equations

$\vec{E}^i(t,x)$ incident electrical wave

$\vec{E}^d(t,x)$ diffracted electrical wave (causal : =0 for $t \leq 0$)

$\vec{E}(t,x)$ total electrical wave

$$\begin{cases} \vec{curl}\vec{E}^d(t,x) + \mu_0 \dfrac{\partial \vec{H}^d}{\partial t}(t,x) = \vec{0} & \text{for } (t,x) \in \mathbb{R} \times \Omega^{ext} \\ \vec{curl}\vec{H}^d(t,x) - \varepsilon_0 \dfrac{\partial \vec{E}^d}{\partial t}(t,x) = \vec{0} & \text{for } (t,x) \in \mathbb{R} \times \Omega^{ext} \\ \vec{E}^d(t,x) \wedge \vec{n}(x) = -\vec{E}^i(t,x) \wedge \vec{n}(x) & \text{for } (t,x) \in \mathbb{R} \times \Gamma \end{cases}$$

b. Integral representation

we prolong the diffracted wave by $-\vec{E}^i$ in Ω. for $t \in \mathbb{R}$ and $x \in \Omega \cup \Omega^{ext}$, we have

$$\left\{ \begin{aligned} \vec{E}^d(t,x) &= \nabla \int_\Gamma \frac{1}{4\pi\varepsilon_0 r} \int_0^{t-r/c} div_\Gamma \vec{j}(\tau, y)\, d\tau d\Gamma_y \\ &\quad - \frac{\partial}{\partial t} \int_\Gamma \frac{\mu_0}{4\pi r} \vec{j}(t - r/c, y)\, d\Gamma_y \\ \vec{H}^d(t,x) &= \vec{curl} \int_\Gamma \frac{1}{4\pi r} \vec{j}(t - r/c, y)\, d\Gamma_y \end{aligned} \right.$$

where $r = |x - y|$, $\vec{j}$ the electrical surface current defined by

$$\vec{j} = [\vec{H}^d \wedge \vec{n}] = (-\vec{H}^i - \vec{H}^d) \wedge \vec{n}$$

c. Variational formulation

$$\left\{\begin{array}{l}
\text{For } \vec{E}^i \wedge \vec{n} \in H_\sigma^{3/2}(\mathbb{R}_+, TH^{-1/2}(div,\Gamma)), \\[2mm]
\text{Find } \vec{j} \text{ in } V = H_\sigma^{1/2}(\mathbb{R}_+, TH^{-1/2}(div,\Gamma)) \text{ such that } \forall \vec{j}^t \in V \\[2mm]
\displaystyle\int_{\mathbb{R}} e^{-2\sigma t} \int_{\Gamma\times\Gamma} \frac{1}{4\pi\varepsilon_0 r} div_\Gamma \left(\int_0^{t-r/c} \vec{j}(\tau,y)\,d\tau \right) div_\Gamma \vec{j}^t(t,x)\; d\Gamma_y d\Gamma_x dt \\[2mm]
\displaystyle -\int_{\mathbb{R}} e^{-2\sigma t} \int_{\Gamma\times\Gamma} \frac{\mu_0}{4\pi r} \frac{\partial \vec{j}}{\partial t}(t - r/c, y).\vec{j}^t(t,x)\; d\Gamma_y d\Gamma_x dt \\[2mm]
\displaystyle = -\int_{\mathbb{R}} e^{-2\sigma t} \int_{\Gamma} \vec{E}^i(t,x).\vec{j}^t(t,x)\; d\Gamma_x dt
\end{array}\right.$$

Existence and Uniqueness (I. TERRASSE).

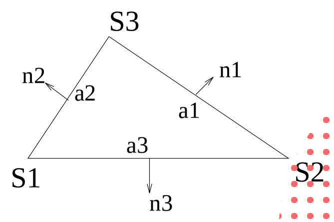
d. Theoretical Ingredients

The proof of the theoretical makes use of the following steps :

- Fourier-Laplace transformation : we obtain a frequency problem with a complex frequency.
- studying the frequency problem : coercivity of the sesqulinear form, continuity estimation with the indexed norms.
- Paley-Wiener theorem : we return to the time domain.

Attention : bilinear form in the time domain is continuous and coercive in different spaces. We can not apply the Lax-Milgram theorem to prove the theoretical results. We study the problem in the frequency domain.

e. Finite Element



$$\vec{j}(t,x) = \sum_{n\geq 1} \vec{j}^n(x)\beta^n(t) = \sum_{n\geq 1} \beta^n(t) \sum_{T\in\tau_h} \sum_{i=1}^{3} \lambda_{i,T}^n \vec{W}_{i,T}(x)$$

$$\vec{j}^t(t_n,x) = \vec{j}^t(x)\beta_0^n(t_n)$$

where

$$\beta^n(t) = \begin{cases} t - t_n & \text{for } t_n \leq t \leq t_{n+1} \\ 0 & \text{for } t < t_{n-1} \\ \Delta t & \text{for } t \geq t_{n+1} \end{cases}$$

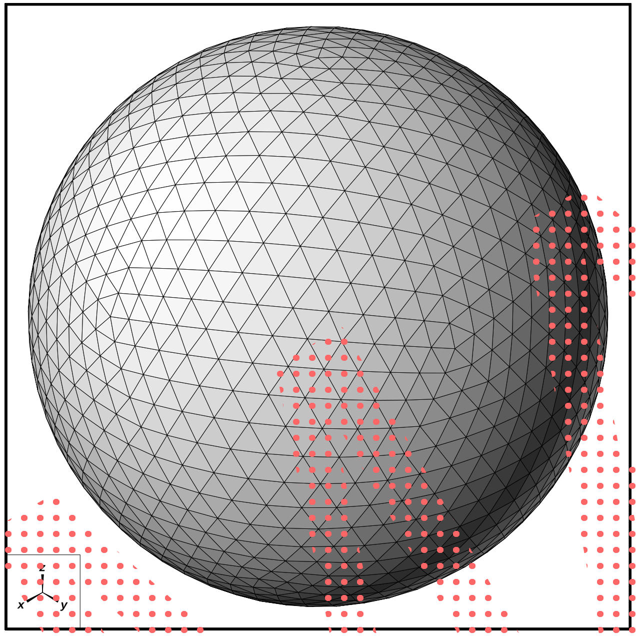
$$\beta_0^n(t_n) = \begin{cases} 1 & \text{for } t_n \leq t \leq t_{n+1} \\ 0 & \text{otherwise} \end{cases}$$

f. Numerical convolution system

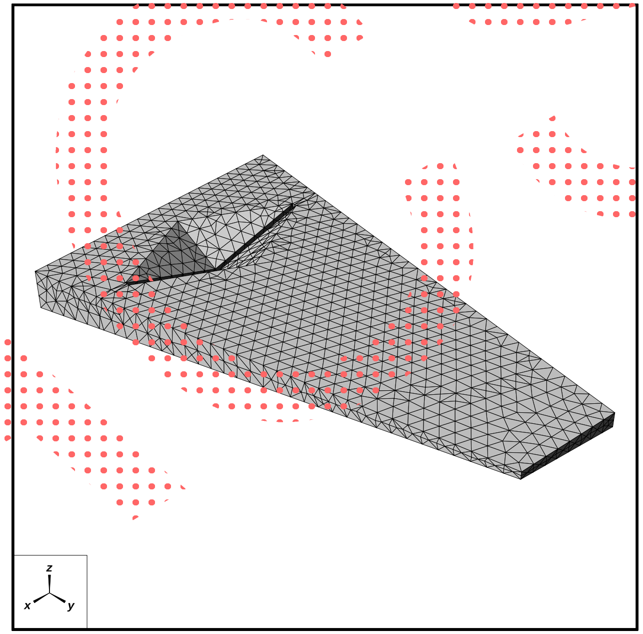
$$\sum_{k=0}^{N} A^k J^{n-k} = b^n$$

- A^k : The matrix of the interaction at distance $k\Delta t$.
- J^{n-k} : The vector of degrees of freedom at time $(n-k)\Delta t$.
- N : The number of matrices determined by Δt and the diameter of the obstacle.

g. Numerical results

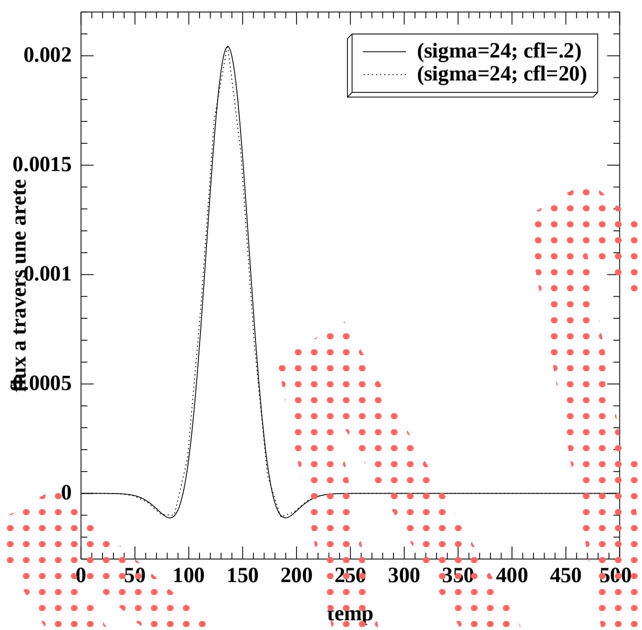


(a) sphere of 2880 triangles : R = 1cm.

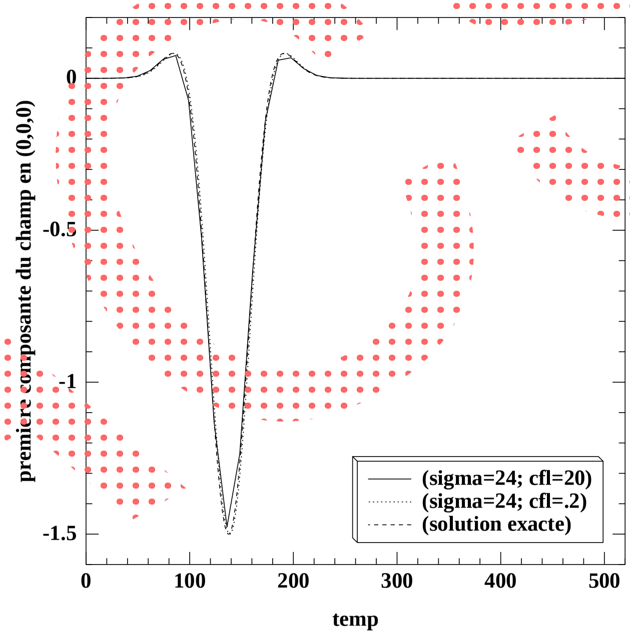


(b) cetaf (Dassault Aviation) of 3594 triangles

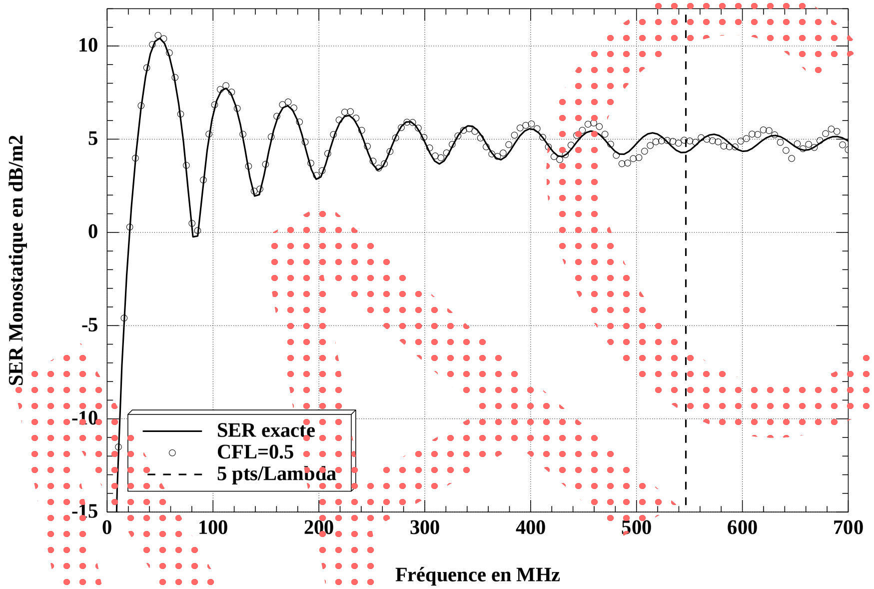
Numerical scheme (Unconditionally stable) : (sphere)



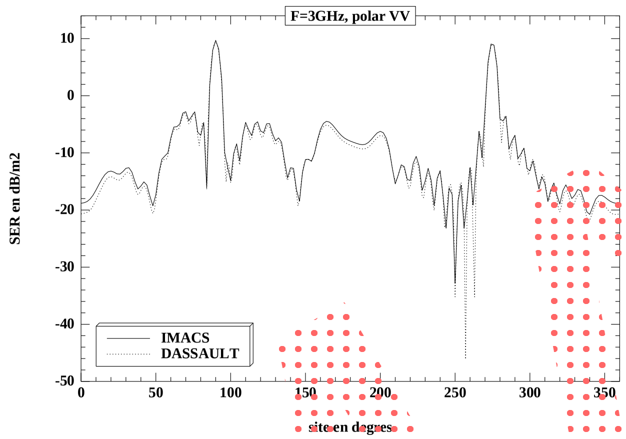
(c) Comparison : flux through an edge (cfl = 0.2 and 20)



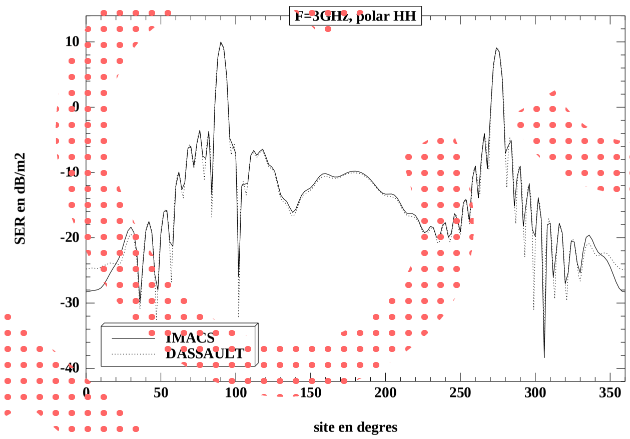
(d) Comparison : solution at (0,0,0) (cfl = 0.2 and 20)



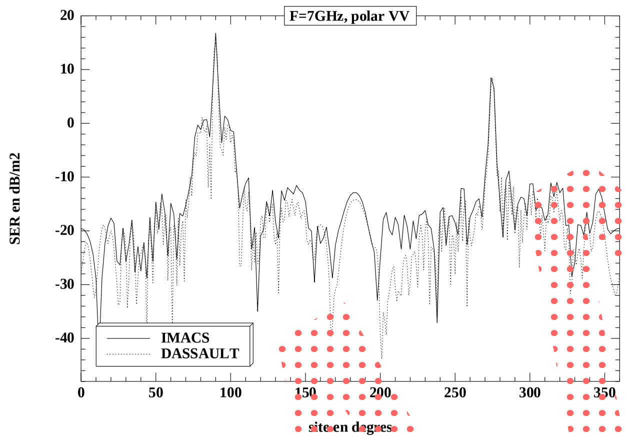
Comparison of the calculated monostatic RCS and the Mie for the sphere (cfl = 0.5)

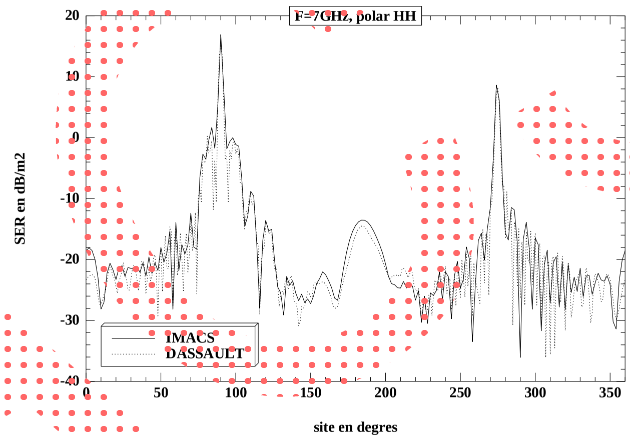


(e) Comparison of the bistatic RCS (VV polarization); freq. = 3Ghz



(f) Comparison of the bistatic RCS (HH polarization); freq. = 3Ghz

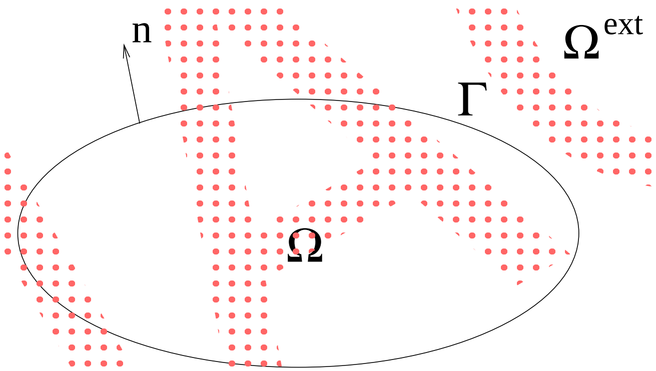




(g) Comparison of the bistatic RCS (VV polarization); freq. = 7Ghz

(h) Comparison of the bistatic RCS (HH polarization); freq. = 7Ghz

2. Generalized impedance conditions



– Virginie LANGE (Z = constant)

a. MAXWELL Equations

$$\begin{cases} \vec{curl}\vec{E}(t,x) + \mu_0 \dfrac{\partial \vec{H}}{\partial t}(t,x) = \vec{0} & \text{for } (t,x) \in \mathbb{R} \times \Omega^{ext} \\ \vec{curl}\vec{H}(t,x) - \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}(t,x) = \vec{0} & \text{for } (t,x) \in \mathbb{R} \times \Omega^{ext} \\ \vec{E}_{||}(t,x) = Z_{t,x}(\vec{H} \wedge \vec{n}) & \text{for } (t,x) \in \mathbb{R} \times \Gamma \end{cases}$$

b. Integral representation

we prolong the diffracted wave $\vec{E}^d$ by $-\vec{E}^i$ in Ω.

$$\begin{aligned}
\vec{E}(t,x) &= \vec{E}^i(t,x) + \vec{\nabla}\int_\Gamma \frac{\int_0^{t-r/c} div_\Gamma \vec{j}(\tau,y)d\tau}{4\varepsilon_0\pi r} d\Gamma_y \\
&\quad -\mu_0\frac{\partial}{\partial t}\int_\Gamma \frac{\vec{j}(t-r/c,y)}{4\mu_0\pi r} d\Gamma_y - c\vec{url}\int_\Gamma \frac{\vec{m}(t-r/c,y)}{4\pi r} d\Gamma_y \\
\vec{H}(t,x) &= \vec{H}^i(t,x) + \vec{\nabla}\int_\Gamma \frac{\int_0^{t-r/c} div_\Gamma \vec{m}(\tau,y)d\tau}{4\mu_0\pi r} d\Gamma_y \\
&\quad -\varepsilon_0\frac{\partial}{\partial t}\int_\Gamma \frac{\vec{m}(t-r/c,y)}{4\pi r} d\Gamma_y + c\vec{url}\int_\Gamma \frac{\vec{j}(t-r/c,y)}{4\pi r} d\Gamma_y
\end{aligned}$$

where $r = |x-y|$, $\vec{j}\ and\ \vec{m}$ the surface currents defined by
$\vec{j} = [\vec{H}^d \wedge \vec{n}] = (-\vec{H}^i - \vec{H}^d) \wedge \vec{n}$ and $\vec{m} = [\vec{n} \wedge \vec{E}] = \vec{E} \wedge \vec{n}$

Definition of some surface operators : (simplify the formulas)

$$\begin{cases} P_t^1(\vec{j}) = \dfrac{1}{\varepsilon_0}\vec{\nabla}_\Gamma \displaystyle\int_\Gamma \frac{\int_0^{t-r/c} div_\Gamma \vec{j}(\tau, y)d\tau}{4\pi r} d\Gamma_y - \mu_0 \Pi_x \frac{\partial}{\partial t}\int_\Gamma \frac{\vec{j}(t-r/c, y)}{4\pi r} d\Gamma_y \\ P_t^2(\vec{j}) = -\Pi_x \displaystyle\int_\Gamma \nabla_x\left(\frac{1}{r}\right) \wedge \left(\vec{j}(t-r/c, y) + \frac{r}{c}\frac{\partial \vec{j}}{\partial t}(t-r/c, y)\right) d\Gamma_y \end{cases}$$

First problem

$$
\text{(S)} \left\{
\begin{array}{ll}
\text{Find } \vec{j} \text{ and } \vec{m} \text{ causal, such that} & \\
-Z_{t,x}(\vec{j}) + \dfrac{\vec{m} \wedge \vec{n}}{2} = \vec{E}_\Gamma^i + P_t^1(\vec{j}) + P_t^2(\vec{m}) & \text{on } \mathbb{R} \times \Gamma \\
\dfrac{\vec{j} \wedge \vec{n}}{2} = \vec{H}_\Gamma^i + \dfrac{\varepsilon_0}{\mu_0} P_t^1(\vec{m}) - P_t^2(\vec{j}) & \text{0n } \mathbb{R} \times \Gamma
\end{array}
\right.
$$

Second problem : we denote $Z_{t,x}^{-1}$ the inverse operator of $Z_{t,x}$

$$
(C)\left\{
\begin{array}{ll}
\text{Find } \vec{j} \text{ and } \vec{m} \text{ causal, such that} & \\
-\dfrac{Z_{t,x}(\vec{j})}{2} = \vec{E}_\Gamma^i + P_t^1(\vec{j}) + P_t^2(\vec{m}) & \text{on } \mathbb{R}\times\Gamma \\
\dfrac{\vec{n}\wedge Z_{t,x}^{-1}(\vec{n}\wedge\vec{m})}{2} = \vec{H}_\Gamma^i + \dfrac{\varepsilon_0}{\mu_0} P_t^1(\vec{m}) - P_t^2(\vec{j}) & \text{on } \mathbb{R}\times\Gamma
\end{array}
\right.
$$

Definition of the bilinear forms b_Γ, b_t^s, b_t^c, and L_t

$$\begin{aligned}
b_\Gamma(\vec{j},\vec{m},\vec{j}^t,\vec{m}^t) &= \langle P_t^1(\vec{j}),\vec{j}^t\rangle_{\sigma,\Gamma} + \langle P_t^2(\vec{m}),\vec{j}^t\rangle_{\sigma,\Gamma} \\
&+ \frac{\varepsilon_0}{\mu_0}\langle P_t^1(\vec{m}),\vec{j}^t\rangle_{\sigma,\Gamma} - \langle P_t^2(\vec{j}),\vec{m}^t\rangle_{\sigma,\Gamma}
\end{aligned}$$

$$\begin{aligned}
b_t^s(\vec{j},\vec{m},\vec{j}^t,\vec{m}^t) &= b_\Gamma(\vec{j},\vec{m},\vec{j}^t,\vec{m}^t) - \frac{1}{2}\langle \vec{j}\wedge\vec{n},\vec{m}^t\rangle_{\sigma,\Gamma} \\
&+ \langle Z_{t,x}(\vec{j}),\vec{j}^t\rangle_{\sigma,\Gamma} - \frac{1}{2}\langle \vec{m}\wedge\vec{n},\vec{j}^t\rangle_{\sigma,\Gamma}
\end{aligned}$$

$$\begin{aligned}
b_t^c(\vec{j},\vec{m},\vec{j}^t,\vec{m}^t) &= b_\Gamma(\vec{j},\vec{m},\vec{j}^t,\vec{m}^t) + \frac{1}{2}\langle Z_{t,x}(\vec{j}),\vec{j}^t\rangle_{\sigma,\Gamma} \\
&+ \frac{1}{2}\langle Z_{t,x}^{-1}(\vec{n}\wedge\vec{m}),(\vec{n}\wedge\vec{m}^t)\rangle_{\sigma,\Gamma}
\end{aligned}$$

$$L_t(\vec{j}^t,\vec{m}^t) = -\langle \vec{E}^i,\vec{j}^t\rangle_{\sigma,\Gamma} - \langle \vec{H}^i,\vec{m}^t\rangle_{\sigma,\Gamma}$$

with $\langle \vec{j},\vec{j}^t\rangle_{\sigma,\Gamma} = \int_{\mathbb{R}}\int_\Gamma e^{-2\sigma t}\vec{j}.\vec{j}^t d\Gamma dt$

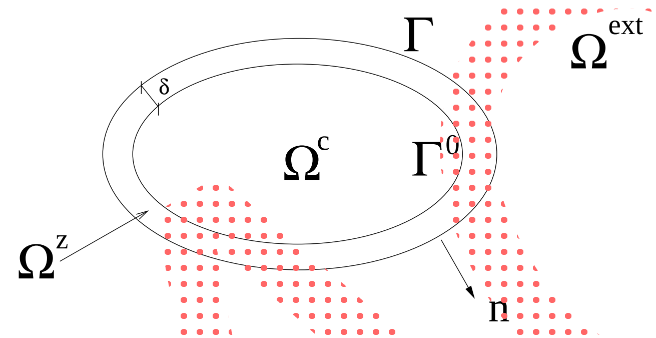
Variational problem associated to (S)

$$(S_v)\begin{cases}\text{Find } (\vec{j},\vec{m}) \in V^s_{t,j}\times V^s_{t,m} \text{ such that } \forall\,(\vec{j}^{\,t},\vec{m}^{\,t}) \in V^s_{t,j}\times V^s_{t,m}\\ b^s_t(\vec{j},\vec{m},\vec{j}^{\,t},\vec{m}^{\,t}) = L_t(\vec{j}^{\,t},\vec{m}^{\,t})\end{cases}$$

Variational problem associated to (C)

$$(C_v)\begin{cases}\text{Find } (\vec{j},\vec{m}) \in V^c_{t,j}\times V^c_{t,m} \text{ such that } \forall\,(\vec{j}^{\,t},\vec{m}^{\,t}) \in V^c_{t,j}\times V^c_{t,m}\\ b^c_t(\vec{j},\vec{m},\vec{j}^{\,t},\vec{m}^{\,t}) = L_t(\vec{j}^{\,t},\vec{m}^{\,t})\end{cases}$$

c. First application : Approximation of “Engquist-Nédélec” for thin layers



– the thickness δ is small.

– ε, μ : the caracteristics of the dielectric layer.

– In frequency domain : $\mathcal{Z}_x(\vec{\mathcal{J}}) = \delta\ \left(-\frac{1}{i\omega\varepsilon}\nabla_\Gamma\ div_\Gamma(\vec{\mathcal{J}}) + i\omega\mu\vec{\mathcal{J}}\right)$

– In time domain : $Z_{t,x}(\vec{j}) = \delta\ \left(\frac{1}{\varepsilon}\nabla_\Gamma\ div_\Gamma \int_0^t \vec{j}(\tau,x)d\tau - \mu\frac{\partial\vec{j}}{\partial t}(t,x)\right)$

Spaces of $\vec{j}, \vec{m}$:

$$\vec{j} \in V^s_{t,j} = H^{1/2}_\sigma(\mathbb{R}_+, TH(div, \Gamma))$$
$$\vec{m} \in V^s_{t,m} = H^{1/2}_\sigma(\mathbb{R}_+, TH^{-1/2}(div, \Gamma))$$

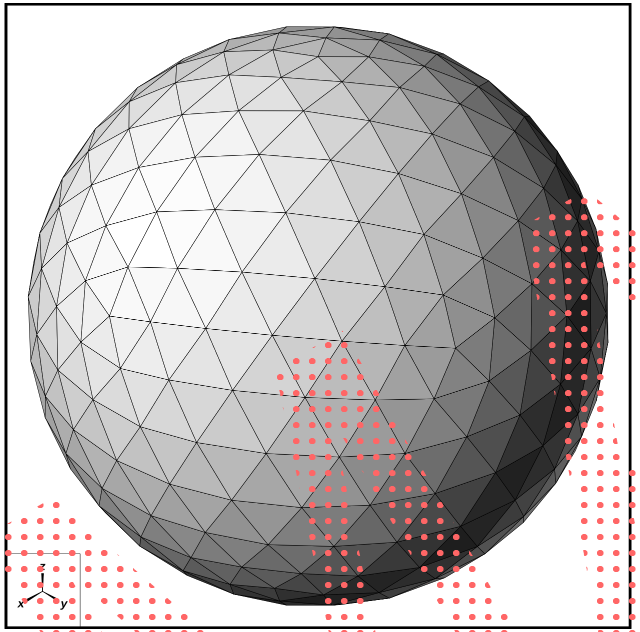
Theorem : The problem (S_v) has a unique solution in $V^s_{t,j} \times V^s_{t,m}$. (Coercivity in the frequency domain transmitted to the time domain).

d. Numerical system (Unconditionally stable)

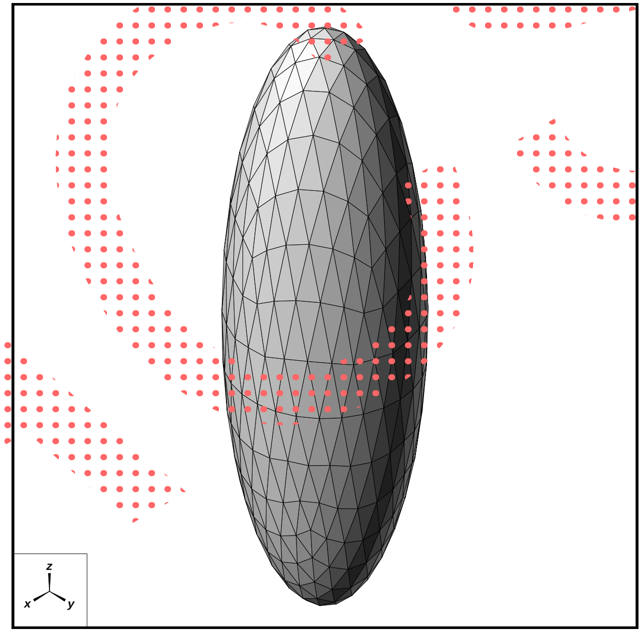
$$\sum_{k=0}^{N} A_1^k J^{n-k} + \sum_{k=0}^{N} C^k M^{n-k} = {b_1}^n$$

$$\sum_{k=0}^{N} A_2^k M^{n-k} + \sum_{k=0}^{N} C^k J^{n-k} = {b_2}^n$$

e. Numerical results

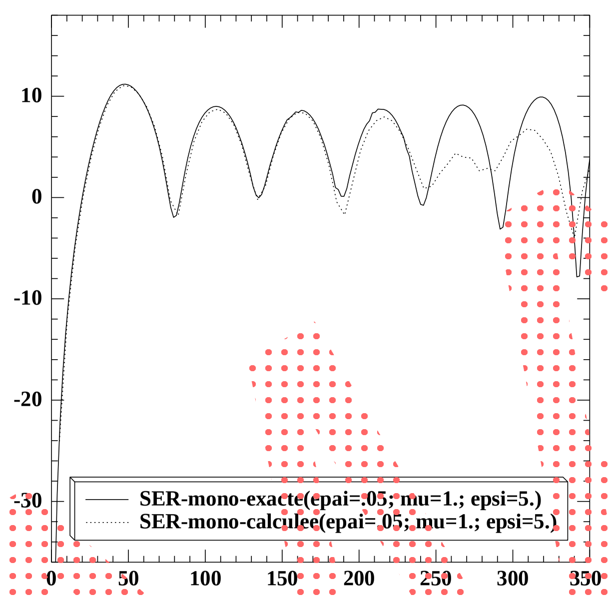


(i) sphere of 720 triangles : R = 1.m.

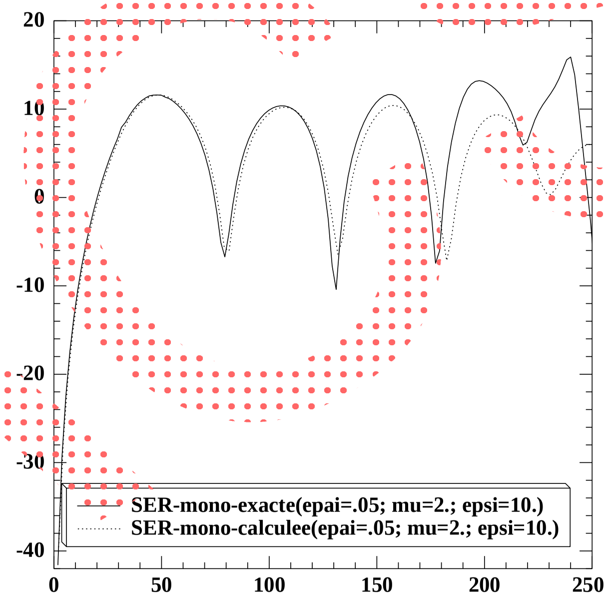


(j) ellipsoid (.3m, .3m,1.m) de 720 triangles

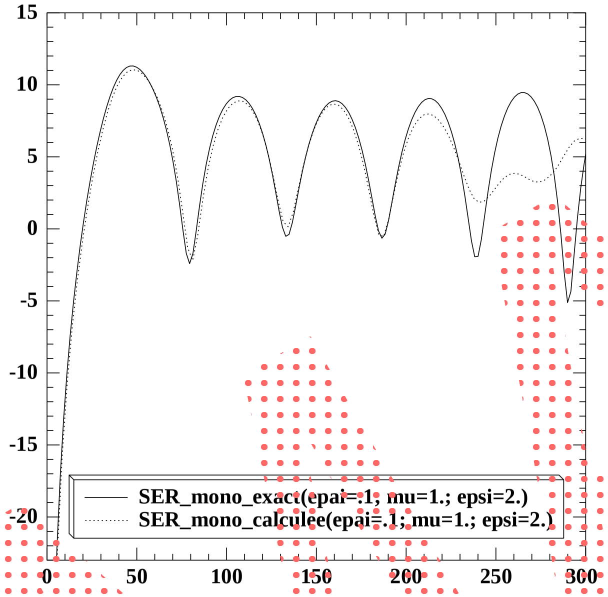
f. Numerical results (sphere)



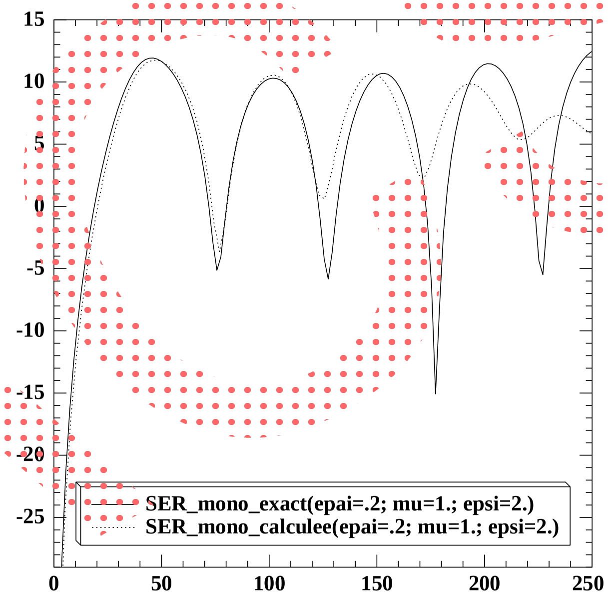
(k) δ=5cm ; ε_r=5 ; μ_r=1 ; f_{5pt} = 118 Mhz



(l) δ=5cm ; ε_r=10 ; μ_r=2 ; $f_{5pt} = 56$ Mhz

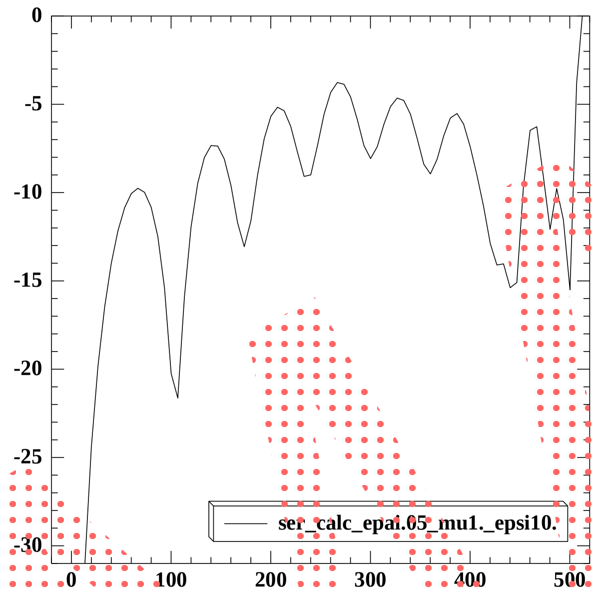


(m) δ=10cm ; ε_r=2 ; μ_r=1 ; $f_{5pt} = 118$ Mhz

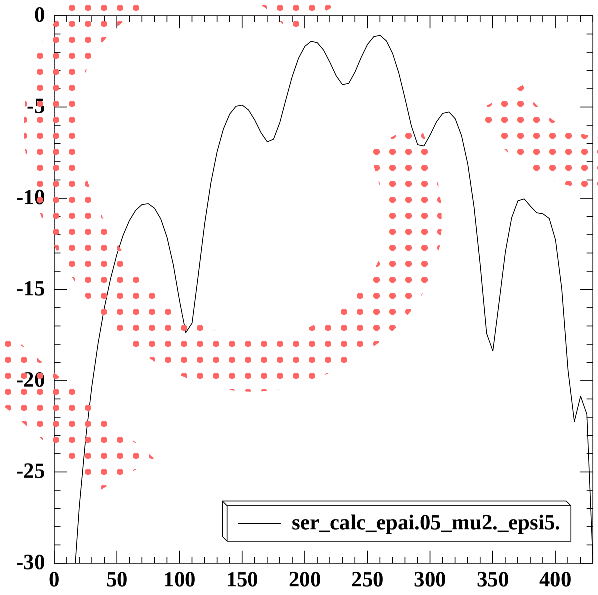


(n) δ=20cm ; ε_r=2 ; μ_r=1 ; $f_{5pt} = 56$ Mhz

g. Numerical results (ellipsoid)

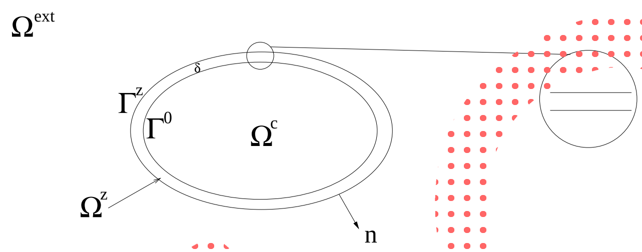


(o) δ=5cm ; ε_r=10 ;
μ_r=1 ; $f_{5pt} = 100$ Mhz



(p) δ=5cm ; ε_r=5 ;
μ_r=2 ; $f_{5pt} = 32$ Mhz

h. Second application : Dielectric thin layer with $\sqrt{\varepsilon_r \mu_r}$ » 1.



- the thickness δ is very small.
- $\sqrt{\varepsilon_r \mu_r}$ » 1.
- in frequency domain : $\mathcal{Z}_x(\mathcal{J}) = i\omega\mu \frac{tg(\alpha\delta)}{\alpha} \vec{\mathcal{J}} \simeq i\omega\mu\delta\vec{j}$; où $\alpha = \frac{\sqrt{\varepsilon_r \mu_r}\omega}{c}$
- in time domain :

$$Z_{t,x}(\vec{j}) = -\delta\, \mu \frac{\partial \vec{j}}{\partial t}(t,x)$$

$$Z_{t,x}^{-1}(\vec{n} \wedge \vec{m}) = -\frac{1}{\delta\, \mu} \int_0^t (\vec{n} \wedge \vec{m}(t,x))dt$$

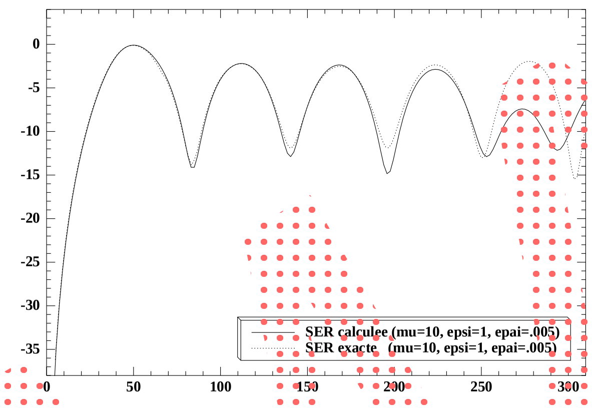
Spaces of $\vec{j}, \vec{m}$:

$$\left\{ \begin{array}{l} V_{t,j} = H_\sigma^{1/2}(\mathbb{R}_+,\, TL^2(\Gamma)) \cap H_\sigma^{1/2}(\mathbb{R}_+,\, TH^{-1/2}(div,\Gamma)) \\ V_{t,m}^s = H_\sigma^{1/2}(\mathbb{R}_+,\, TH^{-1/2}(div,\Gamma)) \\ V_{t,m}^c = H_\sigma^{-1/2}(\mathbb{R}_+,\, TL^2(\Gamma)) \cap H_\sigma^{1/2}(\mathbb{R}_+,\, TH^{-1/2}(div,\Gamma)) \end{array} \right.$$

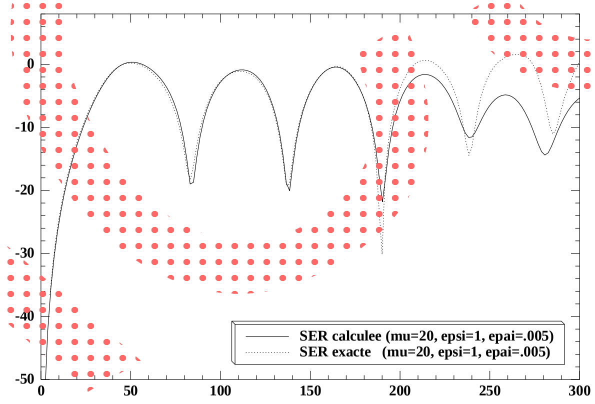
Theorem : The problem (S_v) has a unique solution in $V_{t,j}^s \times V_{t,m}^s$.

Theorem : The problem (C_v) has a unique solution in $V_{t,j}^s \times V_{t,m}^c$.

i. Numerical results (sphere)



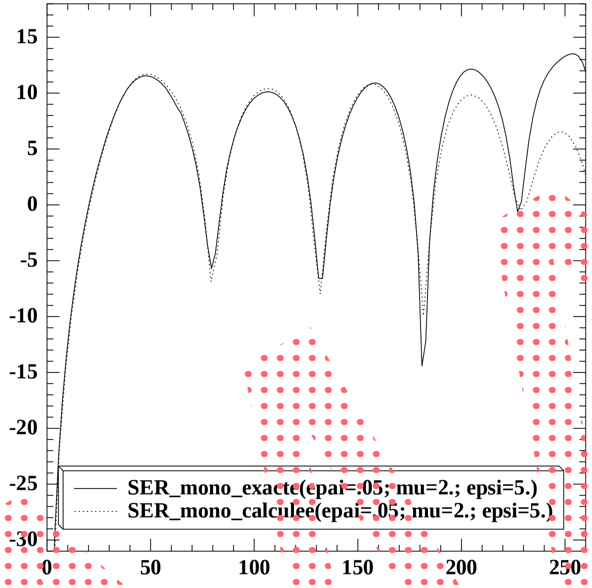
(q) δ=0.5cm ; ε_r=1 ; μ_r=10 ; $f_{5pt} = 118$ Mhz

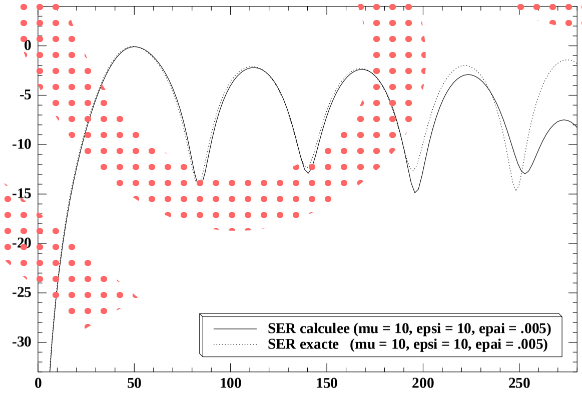


(r) δ=0.5cm ; ε_r=1 ; μ_r=20 ; $f_{5pt} = 56$ Mhz

j. Extension

$$\mathcal{Z}_x(\mathcal{J}) = i\omega\mu\delta(\vec{\mathcal{J}}) - (i\omega)^3\varepsilon\mu^2\delta^3(\vec{\mathcal{J}})$$

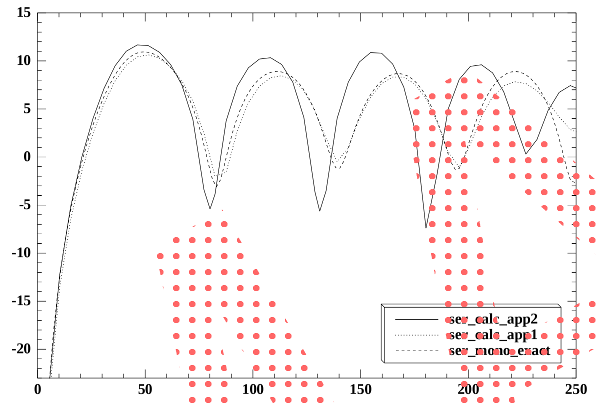




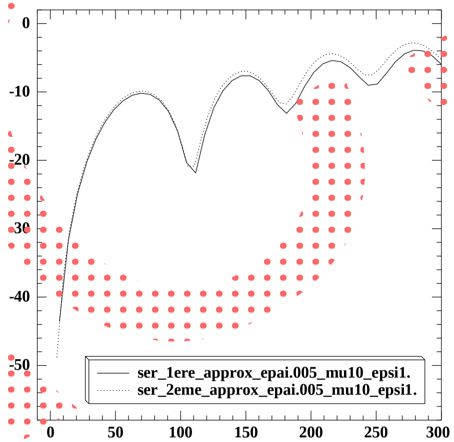
(s) δ=5cm ; ε_r=5 ; μ_r=2 ; $f_{5pt} = 83$ Mhz

(t) δ=0.5cm ; ε_r=10 ; μ_r=10 ; $f_{5pt} = 30$ Mhz

k. Comparison beetween the two applications



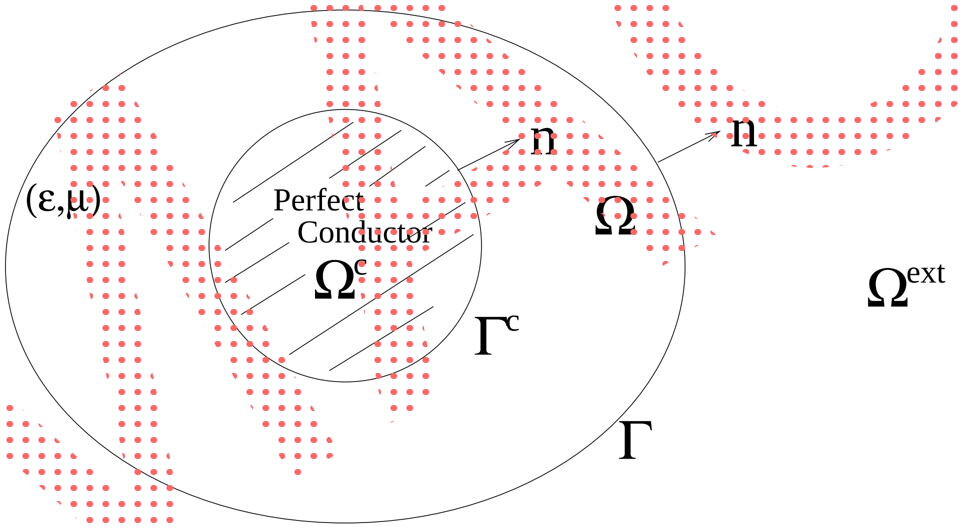
(u) δ=5cm ; ε_r=1 ; μ_r=2 ; f_{5pt} $=$ 185 Mhz



(v) δ=0.5cm ; ε_r=1 ; μ_r=20 ; $f_{5pt} = 83$ Mhz

3. Coupling Maxwell Equations - Retarded Potential Methods

non-homogeneous media :



a. Coupling

Maxwell equations in Ω + Integral representations in Ω^{ext} + continuity of the tangential components.

In frequency domain :

– Vincent LEVILLAIN : Two symetric variationnal formulations (S) and (C). (Existence and uniqueness with $\varepsilon\ and\ \mu$ complex).

In time domain :

– Laurent Bounhoure : parallel work at BORDEAUX

Definition of the bilinear forms b_t^s and b_t^c

$$
\begin{aligned}
b_t^s(\vec{E},\vec{j},\vec{E}^t,\vec{j}^t) &= b_\Gamma(\vec{j},\vec{m},\vec{j}^t,\vec{m}^t) \\
&- \frac{1}{2}\langle \vec{m}\wedge\vec{n},\vec{j}^t\rangle_{\sigma,\Gamma} - \frac{1}{2}\langle \vec{j}\wedge\vec{n},\vec{m}^t\rangle_{\sigma,\Gamma} \\
&+ \langle \varepsilon\frac{\partial\vec{E}}{\partial t},\vec{E}^t\rangle_{\sigma,\Gamma} + \langle \mu\,\vec{curl}\int_R \vec{E}dt,\vec{E}^t\rangle_{\sigma,\Gamma}
\end{aligned}
$$

$$
\begin{aligned}
b_t^c(\vec{E},\vec{H},\vec{E}^t,\vec{H}^t) &= b_\Gamma(\vec{j},\vec{m},\vec{j}^t,\vec{m}^t) \\
&+ \langle \varepsilon\frac{\partial\vec{E}}{\partial t},\vec{E}^t\rangle_{\sigma,\Gamma} + \langle \mu\,\vec{curl}\int_R Edt,\vec{E}^t\rangle_{\sigma,\Gamma} \\
&+ \langle \mu\frac{\partial\vec{H}}{\partial t},\vec{H}^t\rangle_{\sigma,\Gamma} + \langle \varepsilon\,\vec{curl}\int_R \vec{H}dt,\vec{H}^t\rangle_{\sigma,\Gamma}
\end{aligned}
$$

Definition of Spaces :

$$\left\{ \begin{array}{l} H^t = H^1_\sigma(\mathbb{R}_+, H(curl, \Omega)) \\ H^t_c = \left\{ \vec{E} \in H^t;\ such\ that\ \vec{E} \wedge \vec{n} = \vec{0}\ on\ \Gamma^c \right\} \\ V^t = H^{\frac{1}{2}}_\sigma(\mathbb{R}_+, H^{-\frac{1}{2}}(div, \Gamma)) \end{array} \right.$$

Variational Problem (S)

$$(S_v)\left\{\begin{array}{l}\text{Find } (\vec{E},\vec{j}) \in H_c^t \times V^t \text{ such that } \forall\, (\vec{E}^t,\vec{j}^t) \in H_c^t \times V^t \\ b_t^s(\vec{E},\vec{j},\vec{E}^t,\vec{j}^t) = L_t(\vec{m}^t,\vec{j}^t)\end{array}\right.$$

Variational Problem (C)

$$(C_v)\left\{\begin{array}{l}\text{Find } (\vec{E},\vec{H}) \in H_c^t \times H^t \text{ such that } \forall\, (\vec{E}^t,\vec{H}^t) \in H_c^t \times H^t \\ b_t^c(\vec{E},\vec{H},\vec{E}^t,\vec{H}^t) = L_t(\vec{m}^t,\vec{j}^t)\end{array}\right.$$

Existence and uniqueness of the solution

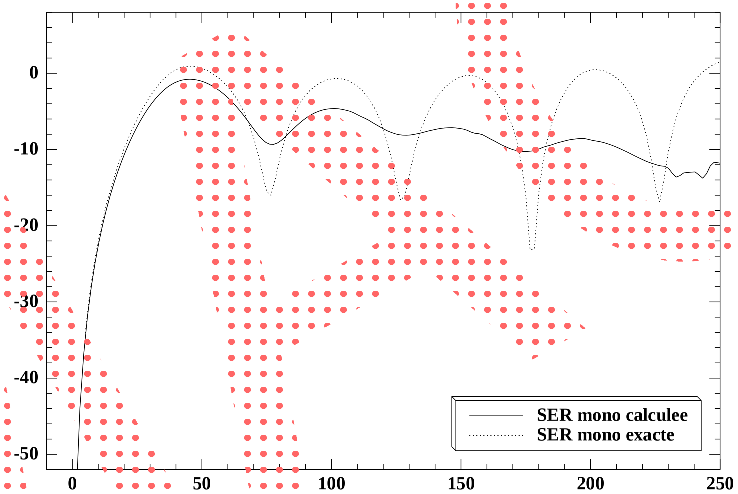
b. Numerical systems and results

$$\sum_{k=0}^{N} A^k \lambda^{n-k} + \sum_{k=0}^{N} C^k \nu^{n-k} - D\nu^n = {b_1}^n$$

$$\begin{aligned}\textstyle\sum_{k=0}^{N} A^k \nu^{n-k} + \sum_{k=0}^{N} C^k \lambda^{n-k} \quad &+ \quad E(S^n - 2S^{n-1} + S^{n-2}) \\ &+ \quad FS^n - D(\lambda^n - \lambda^{n-1}) = {b_2}^n\end{aligned}$$

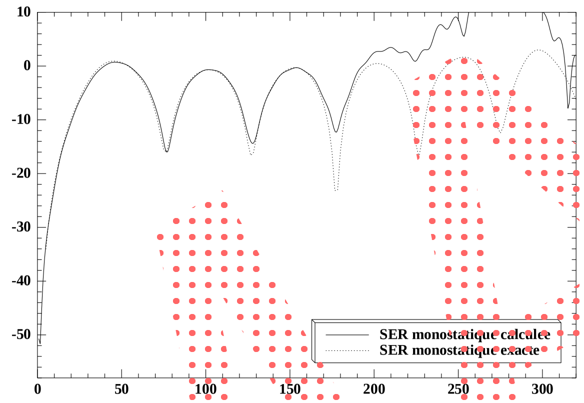
Unconditionally stable but not precise.

Sphere of radius 1m with a cfl=.5, thickness of the layer = .2m, $\varepsilon_r = 2$ and $\mu_r = 1$

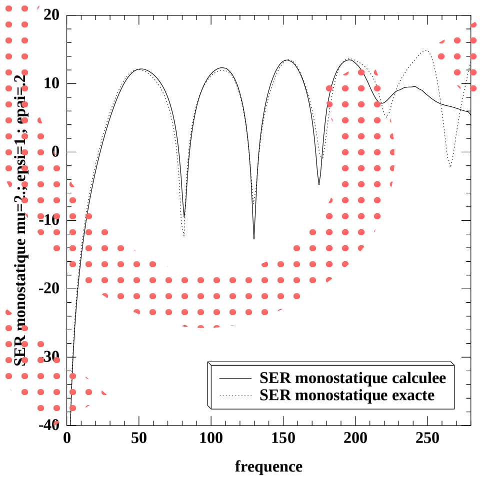


c. Newmark

$\alpha = .25$ and $\beta = .25$

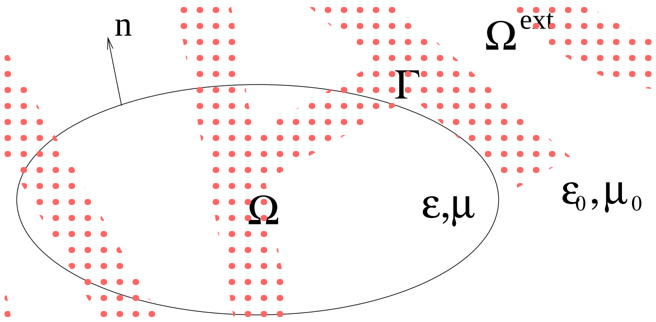


(w) sphere of 620 triangles : R = 1m, $\varepsilon_r = 2.$ and $\mu_r = 1.$.



(x) sphere of 620 triangles : R = 1m, $\varepsilon_r = 1.$ and $\mu_r = 2.$.

4. Retarded Potential for locally homogeneous obstacles.



a. Integral equations

– The total electrical field verifie in Ω^{ext} the previous integral representation with the same surface currents.

– Furthermore, we prolong the total electrical field in Ω by $\vec{0}$ in Ω^{ext}. Thanks to the theorem of integral representation, the electromagnetic field verifies the same representation with the same currents, but with the velocity
$c' = c/\sqrt{\varepsilon_r \mu_r}$.

The continuity of the tangential components define the problem to solve.

b. Variational formulation

We denote by $P^1_{t,c'}$ and $P^2_{t,c'}$ the same operators $P^1_{t,c}$ and $P^2_{t,c}$ but with c'. We introduce

$$\left\{ \begin{array}{l} V = H^{3/2}_\sigma(\mathbb{R}_+,\, TH^{-1/2}(curl,\Gamma)) \\ W = H^{1/2}_\sigma(\mathbb{R}_+,\, TH^{-1/2}(div,\Gamma)) \end{array} \right.$$

and

$$\begin{aligned} b^c_\Gamma(\vec{j},\vec{m},\vec{j}^{\,t},\vec{m}^{\,t}) &= \langle P^1_{t,c}(\vec{j}),\vec{j}^{\,t}\rangle_{\sigma,\Gamma} + \langle P^2_{t,c}(\vec{m}),\vec{j}^{\,t}\rangle_{\sigma,\Gamma} \\ &+ \frac{\varepsilon_0}{\mu_0}\langle P^1_{t,c}(\vec{m}),\vec{j}^{\,t}\rangle_{\sigma,\Gamma^z} - \langle P^2_{t,c}(\vec{j}),\vec{m}^{\,t}\rangle_{\sigma,\Gamma^z} \\ b^{c'}_\Gamma(\vec{j},\vec{m},\vec{j}^{\,t},\vec{m}^{\,t}) &= \langle P^1_{t,c'}(\vec{j}),\vec{j}^{\,t}\rangle_{\sigma,\Gamma} + \langle P^2_{t,c'}(\vec{m}),\vec{j}^{\,t}\rangle_{\sigma,\Gamma} \\ &+ \frac{\varepsilon_0}{\mu_0}\langle P^1_{t,c'}(\vec{m}),\vec{j}^{\,t}\rangle_{\sigma,\Gamma^z} - \langle P^2_{t,c'}(\vec{j}),\vec{m}^{\,t}\rangle_{\sigma,\Gamma^z} \end{aligned}$$

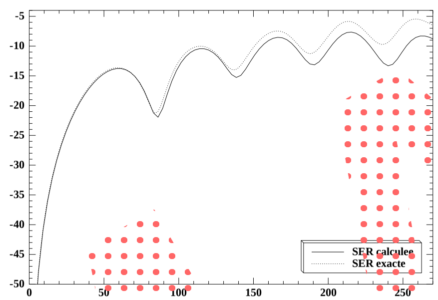
$$b_t(\vec{j},\vec{m},\vec{j}^{\,t},\vec{m}^{\,t}) = b^c_\Gamma(\vec{j},\vec{m},\vec{j}^{\,t},\vec{m}^{\,t}) + b^{c'}_\Gamma(\vec{j},\vec{m},\vec{j}^{\,t},\vec{m}^{\,t})$$

We obtain the variational formulation :

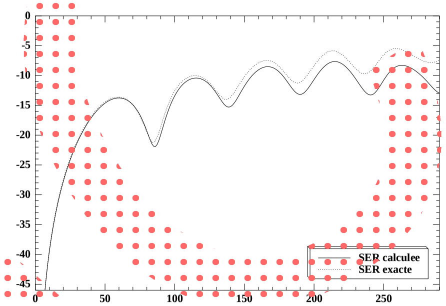
$$(P_v)\left\{\begin{array}{l}\text{For } (\vec{E}^i_\Gamma, \vec{H}^i_\Gamma) \;\in\; V \times V, \\[1em] \text{Find } (\vec{j}, \vec{m}) \in W \times W \text{ such that } \forall\, (\vec{j}^t, \vec{m}^t) \in W \times W \\ b_t(\vec{j}, \vec{m}, \vec{j}^t, \vec{m}^t) = L_t(\vec{j}^t, \vec{m}^t)\end{array}\right.$$

Existence and uniqueness of the solution

c. Numerical results



(y) sphere of 320 triangles : R = 1m, $\varepsilon_r = 2.$ and $\mu_r = 1.$.



(z) sphere of 320 triangles : R = 1m, $\varepsilon_r = 1.$ and $\mu_r = 2.$.

d. General conclusions

Electromagnetic wave diffraction by :

- Perfect conductor obstacles (I.Terrasse and V.Lange).
- Impedance condition : (two practical problems).
- Coupling for non-homogeneous obstacles.
- Locally homogeneous obstacles : (Potential Retarded method)

e. Prospects

- Wires, antennas.
- Resistance, capasitor,...
- Parallelization.
- Vectorization.